國立臺北科技大學九十八學年度碩士班招生考試

系所組別：2310 資訊工程系碩士班甲組

第二節　離散數學與演算法　試題

注意事項：
1. 本試題共十三題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一. (10%) On the set $\boldsymbol{Z}$ of all integers, define the relation $\boldsymbol{R}$ by
$\boldsymbol{R} = \{ (a, b)\in \boldsymbol{Z}\times\boldsymbol{Z} \mid (a^2 - b^2) \text{ is divisible by } 5 \}$.
Show that $\boldsymbol{R}$ is an equivalence relation. Find the equivalence classes of his equivalence relation on $\boldsymbol{Z}$.

二. (15%) For the following problems, please select the correct answers. In each problem, you must have all the correct ones selected to have the full credits. Otherwise, no credits will be given.

(1) (5%) Which of the followings are associative binary operations?
(a)($\boldsymbol{Z}$, *), where $x * y = (x + y) - (x \cdot y)$ for all $x, y \in \boldsymbol{Z}$.
(b)($\boldsymbol{R}$, *), where $x * y = |x + y|$ for all $x, y \in \boldsymbol{R}$.
(c)($\boldsymbol{Z}$, *), where $x * y = \max(x, y)$ for all $x, y \in \boldsymbol{Z}$.

(2) (5%) Which of the followings are monoids?
(a)($\boldsymbol{Z}$, *), where $x * y = y$ for all $x, y \in Z$
(b)($\boldsymbol{Z}$, *), where $x * y = x + y - 3$ for all $x, y \in Z$

(3) (5%) Which of the followings are NOT semigroups?
(a)($\boldsymbol{N}$, *), where $x * y = x^y$ for all $x, y \in \boldsymbol{N}$.
(b)($\boldsymbol{Z}$, *), where $x * y = x$ for all $x, y \in \boldsymbol{Z}$.
(c)($\boldsymbol{Z}$, *), where $x * y = x + y + 2$ for all $x, y \in \boldsymbol{Z}$.

三. (5%) Sheela has six friends. In how many ways can she invite one or more friends to a dinner party?

C^6_1 ~~-1~~

四. (10%) Solve the following recurrence relation
$a_n - 2a_{n-1} - 3a_{n-2} = 5^n,\quad n\geq 2$
with the initial condition $a_0 = -1$ and $a_1 = 1$.

五. (10%) Let $\boldsymbol{L}=\{w\in\{a,b\}^* \mid w=w^R\}$ be a language over the alphabet $\Sigma=\{a, b\}$. Show that $\boldsymbol{L}$ is not a regular language.

六. (5%) Use the master method to give tight asymptotic bounds for the following recurrences. Please justify your answer.（每小題 1 分）

(1) $T(n) = 9T(\frac{n}{3}) + n$

(2) $T(n) = 3T(\frac{n}{4}) + n\lg n$

(3) $T(n) = 4T(\frac{n}{4}) + 3n$

(4) $T(n) = 2T(\frac{n}{3}) + n^{1/3}$

(5) $T(n) = 4T(\frac{n}{2}) + n^2$

七. (5%) What it the asymptotic bound of the following algorithms under the given condition?（每小題 1 分）

(1) What is the running time of HEAPSORT on an array _ of length _ that is already sorted in increasing order?
(2) What is the running time of HEAPSORT on an array _ of length _ that is already sorted in decreasing order?
(3) What is the running time of QUICKSORT when all elements of array A have the same value?
(4) What is the running time of INSERT in a d-ary max-heap in terms of d and n?
(5) What is the running time of RADIXSORT on a total of n d-digit numbers in which each digit can take on up to k possible values?

注意：背面尚有試題

八. (10%) Prove that any comparison sort algorithm requires $\Omega(n \lg n)$ comparisons in the worst case.

九. (5%) Please give a short description to complexity classes P, NP, and NP-complete.

一零.(5%) Give a dynamic-programming solution to the 0–1 knapsack problem that runs in $O(n\ W)$ time, where n is number of items and W is the maximum weight of items that the thief can put in his knapsack.

一一.

1. (5%) What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?

Letter	Frequency
a	1
b	1
c	2
d	3
e	5
f	8
g	13
h	21

2. (5%) Generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

一二. (5%) Give a recursive algorithm Matrix-Chain-Multiply(A, s, i, j) that actually performs the optimal matrix-chain multiplication, given the sequence of matrices $< A_1, \ldots, A_n >$, the s table computed by Matrix-Chain-Order, and the indices i and j. (The initial call would be Matrix-Chain-Multiply(A, s, 1, n))

一三. (5%) Suppose that a graph G has a minimum spanning tree already computed. How quickly can the minimum spanning tree be updated if a new vertex and incident edges are added to G? Please justify your answer.