

國立臺北科技大學九十七學年度碩士班招生考試

系所組別：2310 資訊工程系碩士班甲組

第二節 離散數學與演算法 試題

填准考證號碼

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注意事項：

1. 本試題共十三題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一. (4%) Please test the validity of the logical consequences. You need to show the process for deriving the conclusion.

All polynomials with real coefficients are differentiable functions.

All differentiable functions are continuous.

Therefore, all polynomials with real coefficients are continuous.

二. (4%) Which of the following linear equations has solutions in integers? Please show the reason.

- (a) $28x + 16y = 6$
- (b) $51x + 18y = 12$
- (c) $21x + 35y = 45$

三. (4%) Let S be a finite set with n elements. How many reflexive relations can be defined on S ?

四. (8%) TRUE or FALSE. Please justify the following statements. You do not need to provide the process of derivation.

- (a) (2%) The intersection of two equivalence relations is again an equivalence relation.
- (b) (2%) The union of two equivalence relations is again an equivalence relation.
- (c) (2%) If R_1 and R_2 are two symmetric relations on a set, then so is $R_1 \circ R_2$, where \circ is the composition of two relations.
- (d) (2%) If R is a reflexive and transitive relation, then $R \circ R$ is transitive.

五. (8%) A certain integer between 1 and 1000 leaves the remainder 1, 2, and 7 when divided by 8, 11, and 15, respectively. Please find the integer.

六. (6%) A committee of six is to be made from four students and eight teachers. How many ways can the followings be done?

- (a) (3%) If the committee contains exactly three students?
- (b) (3%) If the committee contains at least three students?

七. (8%) Solve the recurrence $a_n - 3a_{n-1} = 2n$, where $n \geq 1$, with the initial condition $a_0 = 0$.

八. (8%) Show that the language $L = \{a^p \mid p \text{ is a prime integer}\}$ is not a regular language over $\Sigma = \{a, b\}$.

九. What is the asymptotic bound of the following algorithms under the given condition?

- (a) What is the running time of HEAPSORT on an array A of length n that is already sorted in increasing order? (1%)
- (b) What is the running time of QUICKSORT when all elements of array A have the same value? (1%)
- (c) What is the running time of QUICKSORT when the array A contains distinct elements and is sorted in decreasing order. (1%)
- (d) For a given graph $G = (V, E)$, what is the running time of Prim's algorithm using a binary min-heap? (1%)
- (e) For a given graph $G = (V, E)$, what is the running time of Bellman-Ford's algorithm? (1%)

十. A BITONIC-SORTER[n] consists of half-cleaners, which contains $n/2$ comparators and produces two sequences of half the size such that every element in the top half is at least as small as every element in the bottom half. Thus, the sort can be completed by using two copies of BITONIC-SORTER[$n/2$] to sort the two halves recursively. This creates the following recurrence

$$T(n) = \begin{cases} 1 & \text{if } n=2 \\ 2T(\frac{n}{2}) + \frac{n}{2} & \text{if } n=2^k \text{ and } k \geq 2 \end{cases}$$

Find a tight asymptotic bound for the recurrence. Please justify your answer. (5%)

注意：背面尚有試題

十一. A d -ary heap is like a binary heap, but (with one possible exception) non-leaf nodes have d children instead of 2 children.

- (a) How would you represent a d -ary heap in an one-dimensional array A ? (3%)
- (b) How would you map a node with index i to its parent and its j th child (for $1 \leq j \leq d$)? (3%)
- (c) What is the height of a d -ary heap of n elements in terms of n and d ? (3%)
- (d) Write pseudo code for an efficient implementation of EXTRACT-MAX in a d -ary max-heap. Analyze its running time in terms of d and n . (6%)

十二. Let (u, v) be a minimum-weight edge in a graph G . Show that (u, v) belongs to some minimum spanning tree (MST) of G . (10%)

十三.

- (a) Run Dijkstra's algorithm to find the single-source shortest-paths problem on the directed graph of the following figure from the vertex z as the source. Show the d values in the priority queue of vertices and the vertices in set S of vertices whose final shortest-path weights from the source have already been determined after each iteration of the while loop. Please show your work step by step. (5%)
- (b) Write down the pseudo-code of the Dijkstra's algorithm. (5%) What is the running time of Dijkstra's algorithm? Please justify your answer. (5%)

