

國立臺北科技大學九十六學年度碩士班招生考試

系所組別：1820 資訊工程系碩士班乙組

第一節 工程數學 試題

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注意事項：

1. 本試題共 10 題，配分共 100 分，每題 10 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Given the following vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix}$$

express $\mathbf{w} = [2 \ 3 \ 3]^T = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2 + \gamma\mathbf{v}_3$. Find the values of α, β , and γ .

2. Find the rank of the following matrix

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 1 & 1 & 1 \\ 1 & 4 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 1 & 4 \end{bmatrix}.$$

3. A square matrix \mathbf{A} is nilpotent if for some nonnegative integer k , $\mathbf{A}^k = \mathbf{0}_{n \times n}$. Show that \mathbf{A} is nilpotent if and only if all of its eigenvalues are zero.
4. A square matrix \mathbf{A} is called positive definite if $\mathbf{x}^T \mathbf{A} \mathbf{x} > 0$ for all vector $\mathbf{x} \neq \mathbf{0}, \mathbf{x} \in \mathbf{R}^n$. Show that $\det(\mathbf{A}) > 0$.
5. Show that the eigenvectors associated with distinct real-valued eigenvalues of \mathbf{A} are orthogonal if \mathbf{A} is a real symmetric matrix.
6. A communication system transmits binary information over a channel that introduces random bit errors with probability $\varepsilon = 10^{-3}$. The transmitter transmits each information bit three times, and a decoder takes a majority vote of the received bits to decide on what the transmitted bit was. Find the probability that the receiver will make an incorrect

decision.

7. A lot of 40 components are to be inspected. Five components are randomly selected from the lot for inspection. Find the probability that exactly one defective is found in the 5 inspected components if there are 3 defectives in the entire lot.
8. Let Z be the Gaussian random variable with zero mean and unit variance. Find $E[Z^4]$.
9. Let the space of a probability measure be the interval $[0,1]$ and the outcome be equally likely any number ζ in the interval. The random variable X is defined to be $X(\zeta) = 1/\zeta$. Find the density function of the random variable X .
10. Let the random variable Θ be uniformly distributed in the interval $(0, 2\pi)$. Let $X = \cos \Theta$ and $Y = \sin \Theta$. Find $P[X < 0, Y < 0]$.