國立臺北科技大學九十五學年度碩士班招生考試

系所組別：3110、3120、3150 土木與防災研究所甲乙戊組

第二節 工程數學 試題

填准考證號碼

注意事項：
1. 本試題共五題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Given the matrix A as shown.
   (a) Determine the rank of A. (5)
   (b) Is the A matrix singular? (5)
   (c) Determine the eigenvalues of A. (5)
   (d) Determine an eigenvector of A. (5)

$$A=\begin{bmatrix}2 & -3 & 4\\ 1 & 2 & -3\\ 4 & 1 & -2\end{bmatrix}$$

2. Solve the following system of differential equations with $u(0)=v(0)=w(0)=0$ (20)

$$\begin{bmatrix}\frac{du}{dx}\\ \frac{dv}{dx}\\ \frac{dw}{dx}\end{bmatrix}+\begin{bmatrix}1 & -1 & 0\\ 1 & 2 & 1\\ -2 & 1 & -1\end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}e^{2x}$$

3. Solve the following partial differential equation with the boundary conditions of $u(x,\, b)=1$ and $u_x(0,\, y)=u_x(a,\, y)=u(x,\, 0)=0$. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for $0\le x\le a$ and $0\le y\le b$ (20) (Note that you might also need to particularly take care of the case of a zero eigenvalue)

4. (a) Determine the solution of the differential equation $y'+y\tan x=\sin(2x)$ subject to the initial condition of $y(0)=1$. (Note that $\int\tan x dx=-\ln\cos x$) (10)
   (b) Find the general solution of $y''-4y'+4y=0$. (5)
   (c) Find the general solution of $y'''-6y''+11y'-6y=0$. (5)

5. A cantilever beam of length L is subjected to a distributed load $p(x)=x$ as shown in the figure. Using the Euler beam theory, the governing equation is found to be $EIy''''=x$. Apparently, the boundary conditions are $y(0)=y'(0)=y''(L)=y'''(L)=0$. Find the deflection $y(x)$ by Laplace transform. But there is no harm in extending the beam from L to ∞, provided that $p(x)=0$ is defined for $x>L$. Consequently, the problem is to

solve $EIy'''' = x\left[1 - u_L(x)\right]$ with the boundary conditions of $y(0) = y'(0) = y''(L) = y'''(L) = 0$.

Note that $u_L(x) = \begin{cases} 0 & when \quad x < L \\ 1 & when \quad x > L \end{cases}$ where $L \geq 0$, and $\bar{f}(s) = \int_0^\infty f(x) e^{-sx} dx$ where $\bar{f}(s) = \frac{e^{-Ls}}{s}$ for $f(x) = u_L(x)$. (20)