

國立臺北科技大學

九十二學年度資訊工程系碩士班入學考試

工程數學試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共 8 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. (10%) Answer the following statements as true(O) or false(X).
 - (a) If A is a symmetric matrix, then eigenvectors that belong to distinct eigenvalues of A are orthogonal.
 - (b) Let A, B, C be three $n \times n$ matrices. If $AB=AC$, then $B=C$.
 - (c) If A and B are $n \times n$ matrices, then $(A+B)(A+B) = A^2 + 2AB + B^2$.
 - (d) If A and B are both diagonalizable matrices, then $(A+B)^{-1} = A^{-1} + B^{-1}$.
 - (e) If A is an $n \times n$ real matrix and $AA^T = 0$, then $A = 0$.
2. (10%) Show that if A and B are both orthogonal matrices, then AB is also an orthogonal matrix.
3. (15%) Given that A is an $n \times n$ real matrix where n is odd. If A is skew-symmetric (i.e. $A^T = -A$), then $\det(A)=0$.
4. (15%) Let A be an $m \times n$ real matrix with $\text{rank}(A) = n$, and B be an $m \times 1$ real vector. Then derive and find a vector \tilde{x} in R^n to minimize the norm $\|B - A\tilde{x}\|$. Express \tilde{x} by a matrix form in terms of A, B, A^T etc.

5. (10%) Suppose that $n \geq 2$ missiles are fired at a target and hit it independently. If the probability that the i th missile hits it is $p_i, i = 1, 2, \dots, n$, find the probability that at least two missiles will hit the target.
6. (15 %) In a contest, contestants A, B , and C are each asked, in turn, a general scientific question. If a contestant gives a wrong answer to a question, he drops out of the game. The remaining two will continue to compete until one of them drops out. The last person remaining is the winner. Suppose that a contestant knows the answer to a question independently of the other contestants, with probability p . Please calculate the probability of the event E , where $E = \{C \text{ drops out first, } A \text{ next, and } B \text{ wins}\}$.
7. (10%) Let X be a random number from $(0,1)$. Find the probability density function of $Y = -\ln(1 - X)$.
8. (15%) Suppose that 80 balls are placed into 40 boxes at random and independently. What is the expected number of empty boxes?