

國立臺北科技大學九十七學年度碩士班招生考試

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第二節 工程數學 試題

填准考證號碼

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第一頁 共一頁

**注意事項：**

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve the following differential equations

- (a)  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x$  (10%)
- (b)  $\frac{d^2 y}{dx^2} - y = xe^x$  (10%)
- (c)  $\frac{dy}{dx} + (x+1)y = e^{x^2} y^3$  (10%)

2. Solve the following problem by the use of Laplace transforms: (15%)

$$m \frac{d^2 y}{dt^2} + ky = A; \quad y(0) = 0, \quad \frac{dy(0)}{dt} = 0$$

where  $m$ ,  $k$  and  $A$  are constants.

3. If  $\mathbf{r}$  is the position vector  $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ , show that

(a)  $\iiint_S \mathbf{r} \cdot \mathbf{n} dS = 3V$  (5%)

(b)  $\iiint_S x\mathbf{r} \cdot \mathbf{n} dS = 4V \bar{x}$  (5%)

where  $V$  is the volume enclosed by  $S$ ,  $\mathbf{n}$  is the unit outer normal vector and  $\bar{x}$  is the  $x$  coordinate of its center of gravity.

4. Solve the following differential equation by diagonalization. (15%)

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ -6 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ t \end{bmatrix}$$

5. Let  $T$  be the operator on  $\mathbb{R}^2$  defined by  $T(x, y) = (4x-2y, 2x+y)$ ,

- (a) what is the matrix of  $T$  in the ordered basis  $\{[1, 0]^T, [0, 1]^T\}$  (5%)
- (b) what is the matrix of  $T$  in the ordered basis  $\{[1, 1]^T, [-1, 0]^T\}$  (5%)

6. Solve the following partial differential equation (20%)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \text{for } 0 \leq x \leq a, \quad 0 \leq y \leq b \quad \text{and} \quad 0 \leq z \leq c.$$

with the boundary conditions

$$u(0, y, z) = 0, \quad u(a, y, z) = 0, \quad u(x, 0, z) = 0, \\ u(x, b, z) = 0, \quad u(x, y, 0) = 0, \quad u(x, y, c) = f(x, y)$$

where  $a$ ,  $b$  and  $c$  are constants.